

# Light transmission by subwavelength square coaxial aperture arrays in metallic films

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**Abstract:** Using the Fourier modal method, we study the enhanced transmission exhibited by arrays of square coaxial apertures in a metallic film. The calculated transmission spectrum is in good agreement with FDTD calculations. We show that the enhanced transmission can be explained when we consider a few guided modes of a coaxial waveguide.

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**OCIS codes:** (050.1120) Apertures; (240.6680) Surface plasmons; (230.7370) Waveguides

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## References and links

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## 1. Introduction

Nowadays, opticians are greatly interested in structures that exhibit anomalous effects, because they have potential applications in novel photonic devices. The extraordinary enhanced transmission by subwavelength metallic hole arrays is one such phenomenon. Since the publication of by Ebbesen *et al.*,<sup>1</sup> many experimental and theoretical studies were carried out in order to

determine the physical origin of the observed enhanced transmission. Three kinds of explanations have been proposed since, relating the enhanced transmission to the excitation of surface plasmons,<sup>2,3</sup> to a Fabry-Pérot cavity behavior of the holes,<sup>4,5</sup> or explaining the transmission in terms of dynamical<sup>6,7</sup> diffraction. It is now established that both horizontal and vertical resonances play a role<sup>8</sup> in the extraordinary transmission. It is then of importance to characterize and to understand the electromagnetic behavior of the channel through which the light propagates inside the metallic film.<sup>10</sup> Recently, numerical simulations have shown that a transmission as high as 80% can be obtained with anular<sup>11</sup> apertures. The aim of the present communication is to study the spectral response of metallic films with a square coaxial aperture. Those structures are similar to the above-mentioned ones from the electromagnetic point of view. Since the aperture dimensions are of the order of magnitude of the wavelength, a rigorous electromagnetic theory is necessary to analyze the behavior of such structures. Although the FDTD method allows us to calculate rigorously the reflection and transmission of a plane wave by a periodical structure in the resonance domain, the Fourier modal method gives a more physical insight in the present resonant phenomenon.<sup>12</sup> The diffraction problem is reduced to the searching of eigenvalues and eigenvectors of a particular matrix. It permits us to calculate the effective index of the modes of the coaxial aperture and the coupling of these modes with the reflected and transmitted order.

## 2. Statement of the problem

Let us consider a metallic film deposited on a glass substrate with an engraved periodic structure of square coaxial apertures (see Fig. 1).

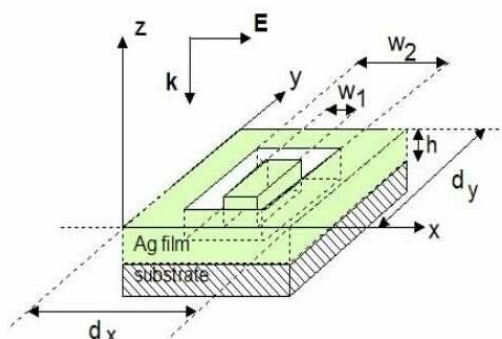


Fig. 1. Coaxial square aperture in a metallic film.

The refractive index of the metal is described by a simple free-electron Drude model with a plasma frequency  $\omega_p = 1.374 \times 10^{16} \text{ s}^{-1}$  and a relaxation time  $\tau = 0.3 \times 10^{-14} \text{ s}$ . The periods are  $d_x$  in the  $x$  direction and  $d_y$  in the  $y$  direction. The width and the position of an aperture are controlled by two parameters  $w_1$  and  $w_2$  (see Fig. 1). Finally, the thickness is denoted by  $h$ . The structure is illuminated in vacuum, under normal incidence, by a monochromatic linearly polarized plane wave, with a wavelength  $\lambda$ , a wavenumber  $k = 2\pi/\lambda$ , and a time dependence  $\exp(i\omega t)$ . Our goal is to calculate and to understand the reflection and transmission spectra of this structure with the help of the Fourier modal method. In the layer, any component  $F$  of the electric or magnetic field can indeed be expressed as a superposition of eigenmodes:

$$F(x, y, z) = \sum_{mnq} (A_q^+ \exp(-ik\gamma_q z) + A_q^- \exp(ik\gamma_q(z-h))) F_{mnq} e_{mn}(x, y)$$

with

$$e_{mn}(x, y) = \exp\left(-i\frac{2m\pi x}{d_x}\right) \exp\left(-i\frac{2n\pi y}{d_y}\right)$$

where  $m$  and  $n$  are integers such that  $-M \leq m \leq M$  and  $-N \leq n \leq N$ . The integers  $M$  and  $N$  describe the truncation scheme. The matrix from which eigenvalues and eigenvectors are calculated is then of rank  $2(2M+1)(2N+1)$ .  $A_q^+$  and  $A_q^-$  are the unknown complex amplitudes of the upward and downward propagating or decaying waves. Our numerical code includes the correct factorization rules derived by Li,<sup>9</sup> our personal parametric formulation, and the S matrix approach for writing the boundary conditions. It should be emphasized that the above-mentioned numerical tools are of great importance for obtaining reliable and converged results, although this point is beyond the scope of our paper. To compare the Fourier modal method and the FDTD that was used by Baida and Van Labeke,<sup>11</sup> we have calculated the transmission spectrum of a structure with the following parameters:  $w_1 = 105 \text{ nm}$ ,  $w_2 = 155 \text{ nm}$ ,  $d_x = d_y = 300 \text{ nm}$ ,  $h = 150 \text{ nm}$ ,  $n_s = 1.45$ .

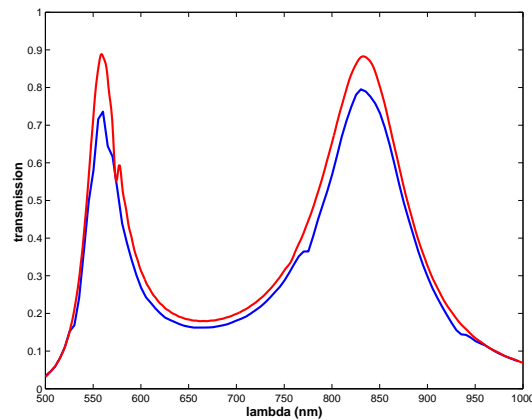


Fig. 2. Transmission of a square coaxial aperture calculated with the FDTD (blue curve) and the Fourier modal method (red curve).

It can be seen that both methods give resonances at the same place even though a small difference is observed in their intensity.

### 3. Discussion

#### 3.1. Analysis of the mode

Our goal is to analyze the enhanced transmission by using the guided modes of the coaxial apertures. Since we consider a metallic medium, an aperture is not coupled with its neighbors. A mode for the entire structure thus corresponds exactly to a mode of a sole aperture, and thus no distinction is made in this discussion between them. Indeed, the eigenvalues and the fields inside the apertures corresponding to an eigenmode do not change when the distance between holes varies. As a consequence, the eigenvalues  $\gamma_q$  give an immediate access to the effective index for each guided mode.

Because of the metal, all the propagating constants are complex but some of them can be considered as guided modes with low losses. For the considered structure we have found that there were three such modes, two of them being degenerated as a result of the square symmetry. The numerically obtained dispersion relations are plotted in Figs. 3 and 4.

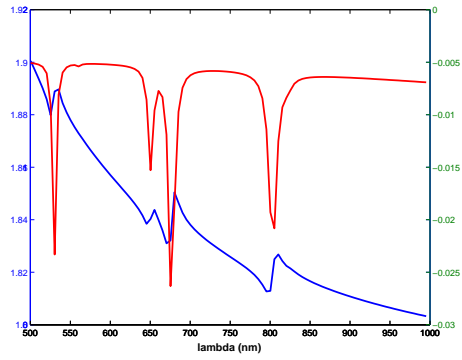


Fig. 3. Dispersion curves of the first mode. Blue curve, real part; red curve, imaginary part. The presence of dips is probably due to the right angle corners.

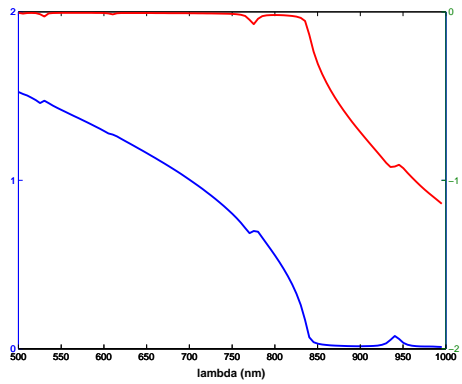


Fig. 4. Dispersion curves of the second mode. Blue curve, real part; red curve, imaginary part.

Figures 5 and 6 show a map of the modulus of the transverse electric field of the first and the second modes.

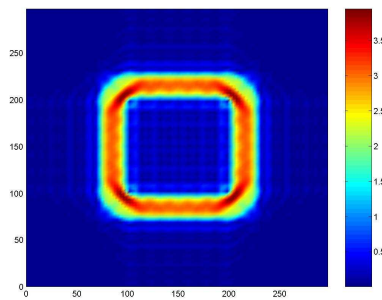


Fig. 5. Modulus of the transverse electric field of the first guided mode.

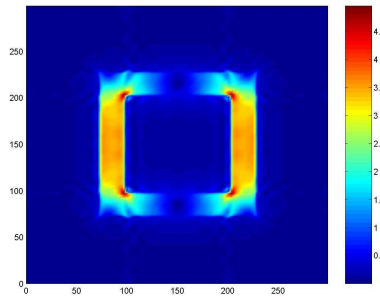


Fig. 6. Modulus of the transverse electric field of the second guided mode.

The mode whose effective index has the largest real part and the lowest imaginary part corresponds to the TEM mode of the same coaxial structure with perfect conducting walls. This mode is characterized by an electric field normal to the walls and has no cut-off. In the present case, it is not strictly speaking a TEM mode since its effective index is greater than one. However, when the width of the aperture becomes larger, the coupling between the opposite sides of the coaxial waveguide diminishes resulting in a lower effective index. The two other guided modes have a cut-off  $\sim \lambda = 845 \text{ nm}$ .

### 3.2. Analysis of the coupling of the modes to free radiation

We have shown the existence of attenuated guided modes. In the present section we are interested in the way they can be excited by an incident plane wave. The S matrix approach is a very appropriate tool for such an analysis. Let us consider the particular wavelength  $\lambda = 558 \text{ nm}$  where a resonance occurs. Figures 7 and 8 show the 21st calculated modal coefficients corresponding to the upward and downward waves inside the aperture when the film is illuminated by an x-polarized plane wave under normal incidence.

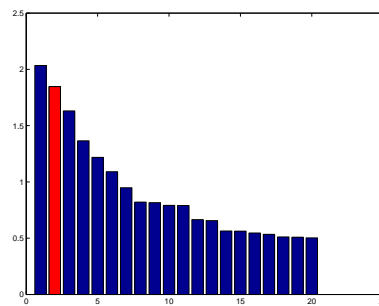


Fig. 7. Twenty-first modal amplitude coefficients inside the coaxial on the upper face. The red bar corresponds to an attenuated guided wave.

For convenience, sorting is displayed in decreasing order. To obtain some physical information from this spectrum analysis, we have carefully normalized all the eigenvectors. It should be noted that the above coefficients are calculated on the interface where the corresponding wave has been excited. By considering the eigenvalues, i.e., the normalized propagating constants, one can easily deduce which kind of mode is excited. Figure 9 represents the location in the

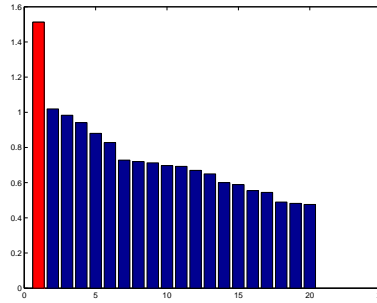


Fig. 8. Twenty-first modal amplitude coefficients inside the coaxial on the lower face. The red bar corresponds to an attenuated guided wave.

complex plane of the propagating constant associated to the modal amplitude of Fig. 7. In Fig. 7, the first and the third modes are a degenerated mode whose the imaginary part of the effective index is as high as 22.8. In the present case, we can conclude that the mode responsible for the resonant transmission is the attenuated guided mode that matches the polarization of the incident wave. This mode has an effective index of  $1.39 - 0.006i$ .

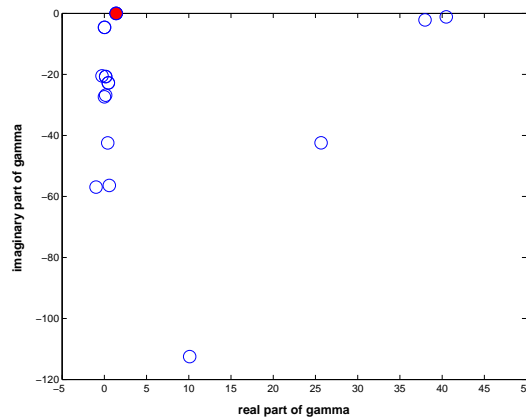


Fig. 9. The ten first complex propagating constants associated to the ten first modes that are exited on the upper face inside the coaxial waveguide. The red one corresponds to an attenuated guide wave; its value is  $\gamma = 1.39 - 0.006i$ .

#### 4. Conclusion

We have numerically studied the spectral response of subwavelength coaxial apertures. We have calculated the propagating constants of the modes supported by a square coaxial waveguide. Some of them correspond to attenuated guided modes. However, the excitation of such modes is possible only when the incident wave matches the mode profile. Owing to the electric properties of metals at optical wavelengths, the dispersion relations of the modes of the transmission channel are very specific and very different from those of the same channel with perfectly conducting walls. This preliminary study paves the way for future investigations in order to engineer the modes and their excitation for applications.